DYNAMICAL MODELS OF REPEATED GOAL-DIRECTED MOVEMENTS

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INTRODUCTION

A class of discrete dynamical systems for modeling repeated goal-directed kinematically redundant human movements is presented. The approach is based on the definition of movement tasks using goal functions and Goal Equivalent Manifolds (GEMs) (Cusumano and Cesari, 2006), which helps us to address "Bernstein's degrees of freedom problem". The example of a ball-throwing task is used to illustrate the central ideas. The resulting perception-action models have a hierarchical structure involving in-trial action templates and an inter-trial stochastic optimal error correction. The models identify the important factors on which variability in repetitive skilled performance depends.

THROWING TASK EXAMPLE

Consider a ball-throwing task as is schematically shown in Fig. 1. The manipulandum starts from rest ($\theta = 0, \omega = 0$) and throws a ball at the target on the wall. Knowing (θ, ω) at release completely determines if the ball hits the target. The scalar goal function for this task in the k^{th} trial can be written as,

$$f(\theta_k, \omega_k; H) = \frac{L + R\cos\theta_k}{\tan\theta_k} - \frac{g}{2}\left(\frac{L + R\cos\theta_k}{\omega_k R\sin\theta_k}\right)^2 + R\sin\theta_k - H = 0, \quad (1)$$

The set of all $\mathbf{x} = (\theta, \omega)$ that satisfies Eq.(1) constitutes the goal equivalent manifold (GEM). In order to attain the release values (θ, ω) the manipulandum has to be actuated from rest. Experimental observations suggest that optimization principles govern discrete movements (Engelbrecht, 2001). Here, for the sake of simplicity, we assume that the manipulandum follows an *in-trial action template*

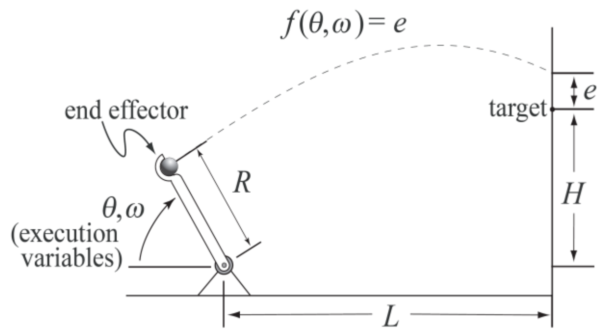

Figure.1 A Ball-throwing task. The goal is to hit the target at height H .

given by a minimum mean squared torque path such that, $\delta \int_0^{\tau_k} \left(\ddot{\theta}\right)^2 \mathrm{d}\tau = 0$. This transforms the execution variables to the action variables as: $\theta_k = \frac{1}{6}\beta_k \tau_k^3$; $\omega_k = \frac{1}{2}\beta_k \tau_k^2$, where β is a jerk parameter and τ is the rescaled dimensionless actuation time to release. We can now equivalently express the GEM for the throwing task in terms of the actuation variables as:

$$f\left(\theta_k, \omega_k\right) = f\left(\tfrac{1}{6}\beta_k \tau_k^3 \; \tfrac{1}{2}\beta_k \tau_k^2\right) \triangleq \tilde{f}\left(\beta_k, \tau_k\right).$$

Figure 2 shows the GEM in action space with two constant error contours depicting $\pm 10\%$ error at the target. We notice that the "thickness" imparted to the GEM by these constant error contours is not constant and varies along the GEM. For small perturbations off of the GEM, the singular values s of the matrix of partial derivatives of the goal function with respect to the body variables (A), quantify the effective thickness property of the GEM and determine how body errors are magnified to goal level errors. This property of the goal function is referred to as *passive sensitivity* and is central to our analysis of variability arising in repeated trials of skilled tasks.

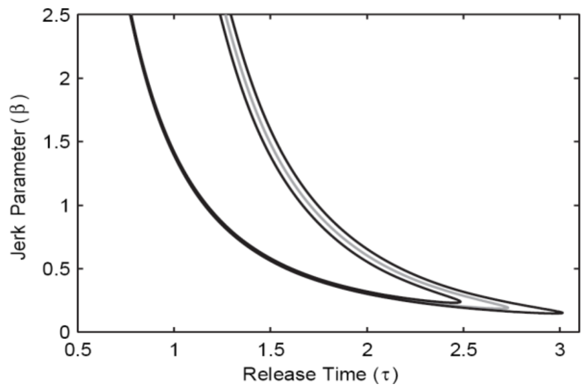

Figure 2. The GEM (grey) and constant error $(\pm 10\% \,\text{error})$ contours (black) in the actuation space for the throwing task.

PERCEPTION-ACTION LOOP

The schematic representation in Fig. 3 is used to illustrate the repeated performance of a goal oriented task. This template is fairly general and can be used to model a variety of repetitive skilled tasks. The inter-trial dynamics can be modeled as a simple update equation :

$$\mathbf{x}_{k+1} = \mathbf{x}_k + (G + M)\ \mathbf{v}(\mathbf{x}_k) + \mathbf{n}, \qquad (2)$$

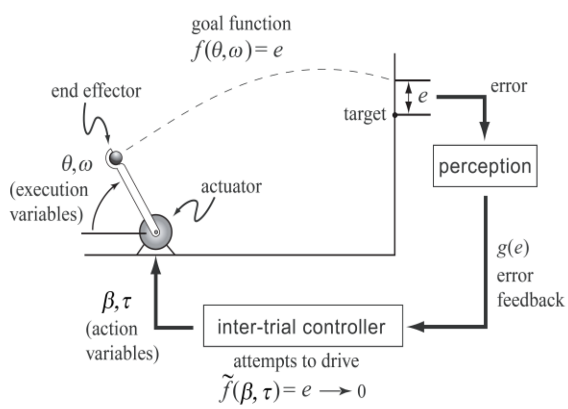

Figure 3. The perception-action loop. Each component in this template can be modified according to the task being modeled.

where $\mathbf{x}_k = (\beta_k, \tau_k)$ are the body state variables in the action space, in the k^{th} trial, $\mathbf{v}(\mathbf{x}_k)$ is the control input, $\mathbf{n}$ is the additive noise and G and M are diagonal matrices of controller gain and multiplicative noise respectively. An error-correcting stochastic optimal feedback controller (Todorov & Jordan, 2005) is employed to compute the input $\mathbf{v}(\mathbf{x}_k)$. For small deviations $\mathbf{u}$ from the GEM, Eq.(4) is linearized as $\mathbf{u}_{k+1} = (I + GJ)\,\mathbf{u}_k + \mathbf{n}$, where J is the Jacobian matrix of the control input $\mathbf{v}$ evaluated at $\mathbf{x}^*$. The eigenvalues of the matrix $(I + GJ)$ determine the stability of the system. Transforming the linearized dynamics to the eigen coordinates, it can be shown that the performance at the target scales linearly with the passive sensitivity of the GEM, i.e.,

$$\frac{\sigma_e}{\sigma} = \frac{s}{\sqrt{1-\lambda^2}}\,, \qquad (4)$$

where σ_e^2 is the variance of error at the target, σ^2 is the variance of the isotropic additive noise, s passive sensitivity, and λ is the eigenvalue of the linearized controller corresponding to a direction orthogonal to the GEM. Repetitive performance based on Eq.(2) at 20 different locations along the GEM, for two different controllers (optimal: $\lambda = 0,$ suboptimal: $\lambda = 0.6)$ and two different additive noise levels ($\sigma = 10^{-4}$ and $\sigma = 10^{-2}$), was numerically simulated. The variance at the target (σ_e) was found to agree very well with the prediction of Eq.(4).

SUMMARY

We showed how the concepts of goal equivalence and goal equivalent manifolds can be used to developed a class of models for the trial-to-trial dynamics of skilled movements. Applying the approach to a throwing example, we found that the performance at the target depends on passive sensitivity (s), noise level (σ), and the stability of the inter-trial controller (λ). Experiments to test these predictions are currently being constructed.

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